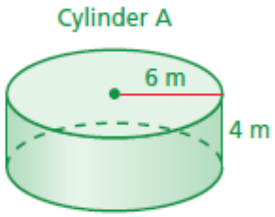
Lesson 8.4:

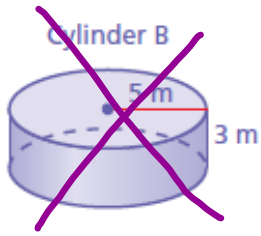
Surface Areas and Volumes of Similar Solids

Essential Question

When the dimensions of a solid increase by a factor of *k*, how does the surface area change? How does the volume change?

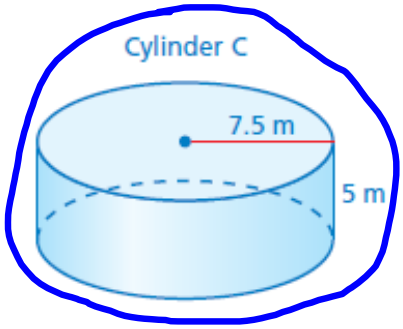
Which cylinder is similar to Cylinder A?





$\frac{6}{5} \times \frac{4}{3}$

$18 \neq 20$

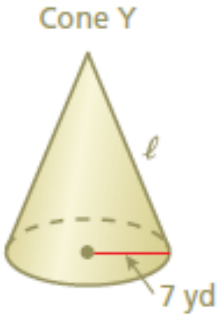


$\frac{6}{7.5} \times \frac{4}{5}$

$30 = 30$ ✓

Example 1

The cones are similar. Find the missing slant height ℓ.



$\frac{5}{7} \times \frac{13}{\ell}$

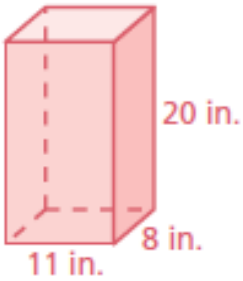
$5 \cdot \ell = 91$

$\div 5 \quad \div 5$

$\ell = 18.2$ yd

Example 2

The prisms at the right are similar. Find the missing width and length.



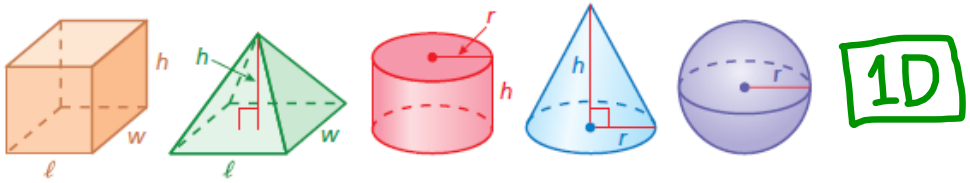
$\frac{20}{8} \times \frac{8}{w}$ $20w = 64$ $\div 20 \quad \div 20$ $w = 3.2$ in

$\frac{20}{8} \times \frac{11}{\ell}$ $20 \cdot \ell = 88$ $\div 20 \quad \div 20$ $\ell = 4.4$ in

On your own 1-2

Key Ideas

Linear Measures

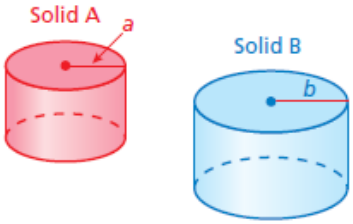


1D

Surface Areas of Similar Solids

When two solids are similar, the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.

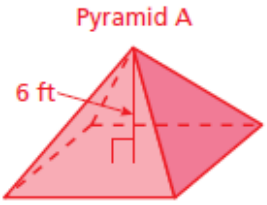
$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{a}{b}\right)^2$$

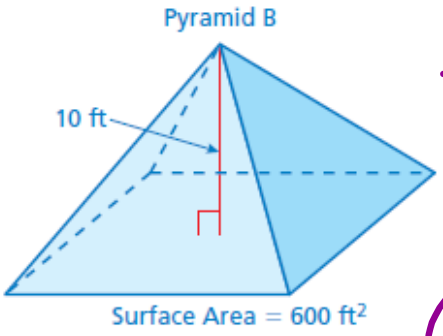


2D

Key Idea

The pyramids are similar. What is the surface area of Pyramid A?





$$\frac{SA_A}{600} = \left(\frac{6}{10}\right)^2$$

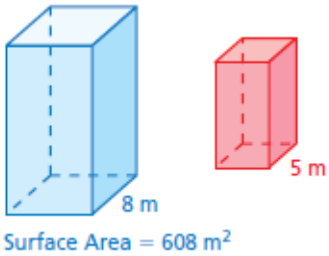
$$\frac{SA_A}{600} = \frac{36}{100} \quad (\cdot 6)$$

$$SA_A = 216\,ft^2$$

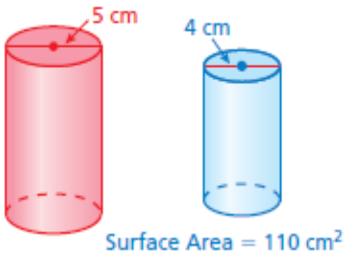
Example 3

The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

3.



4.



$$\frac{SA}{608} = \left(\frac{5}{8}\right)^2$$

$$\frac{SA}{608} \times \frac{25}{64}$$

$$SA \cdot 64 = 15200$$

$$SA = 237.5\,m^2$$

$$\frac{SA}{110} = \left(\frac{5}{4}\right)^2$$

$$\frac{SA}{110} \times \frac{25}{16}$$

$$SA \cdot 16 = 2750$$

$$SA = 171.9\,cm^2$$

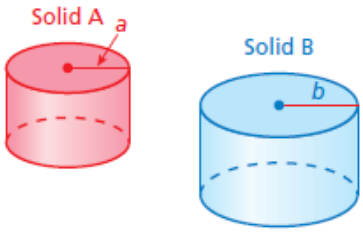
On your own 3-4

Key Idea

Volumes of Similar Solids

When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{a}{b}\right)^3$$



3D

Key Idea

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

3D

Original Tank

Volume = 2000 ft^3

(A) 150 ft^3 (B) 4000 ft^3

(C) 8000 ft^3 (D) 16,000 ft^3

$$\frac{V}{2000} = \left(\frac{2}{1}\right)^3$$

$$\frac{V}{2000} = 8$$

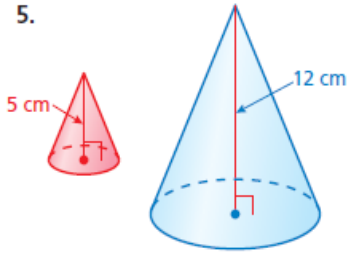
$\times 2000$ $\times 2000$

$V = 16{,}000$ ft^3

Example 4

The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.

5.



Volume = 288 cm^3

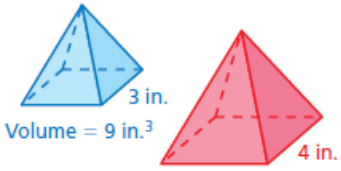
$\frac{V}{288} = \left(\frac{5}{12}\right)^3$

$\frac{V}{288} \times \frac{125}{1728}$

$1728V = 36000$

$V \approx 20.8 \text{ cm}^3$

6.



Volume = 9 in.^{3}

$\frac{V}{9} = \left(\frac{4}{3}\right)^3$

$\frac{V}{9} \times \frac{64}{27}$

$27V = 576$

$\div 27 \quad \div 27$

$V \approx 21.3 \text{ in}^3$

On Your Own 5-6