

7.4 Extension:

Repeating Decimals

Key Idea

Writing a Repeating Decimal as a Fraction

Let a variable x equal the repeating decimal d .

Step 1: Write the equation $x = d$.

Step 2: Multiply each side of the equation by 10^n to form a new equation, where n is the number of repeating digits.

Step 3: Subtract the original equation from the new equation.

Step 4: Solve for x .

Write $0.\overline{4}$ as a fraction in simplest form.

$$\begin{array}{r}
 x = 0.\overline{4} \\
 \cdot 10 \quad \cdot 10 \\
 \hline
 10x = 4.\overline{4} \\
 -(x = 0.\overline{4}) \\
 \hline
 9x = 4 \\
 \div 9 \quad \div 9 \\
 \hline
 x = \boxed{\frac{4}{9}}
 \end{array}$$

Write $-0.\overline{23}$ as a fraction in simplest form.

$$\begin{array}{r}
 x = -0.\overline{23} \\
 \cdot 100 \quad \cdot 100 \\
 \hline
 100x = -23.\overline{23} \\
 -(x = -0.\overline{23}) \\
 \hline
 99x = -23 \\
 \div 99 \quad \div 99 \\
 \hline
 x = \boxed{-\frac{23}{99}}
 \end{array}$$

Write $1.\overline{25}$ as a mixed number.

$$\begin{array}{r|l}
 X = 1.\overline{25} & \\
 \cdot 100 & \cdot 100 \\
 \hline
 100x = 125.\overline{25} & \\
 \hline
 x = 1.\overline{25} & \\
 \hline
 99x = 124 & \\
 \frac{99}{99} & \frac{124}{99} \\
 \hline
 X = \frac{124}{99} = 1\frac{25}{99} &
 \end{array}$$

Write $0.2\overline{3}$ as a fraction.

$$\begin{array}{r|l}
 X = 0.2\overline{3} & \\
 \cdot 10 & \cdot 10 \\
 \hline
 10x = 2.\overline{3} & \\
 \hline
 x = 0.2\overline{3} & \\
 \hline
 9x = 2.1 & \\
 \frac{9}{9} & \frac{2.1}{9} \\
 \hline
 X = \frac{2.1}{9} &
 \end{array}$$

$$\begin{array}{l}
 \frac{2.1 \cdot 10}{9 \cdot 10} \\
 \frac{21 \div 3}{90 \div 3} \\
 \frac{7}{30}
 \end{array}$$

Write $2.\overline{35}$ as a mixed number.

$$\begin{array}{r} x = 0.\overline{35} \\ \cdot 100 \quad \cdot 100 \\ \hline 100x = 35.\overline{35} \\ \leftarrow (x = 0.\overline{35}) \\ \hline 99x = 35 \\ \frac{99}{99} \quad \frac{35}{99} \\ x = \frac{35}{99} \end{array} \rightarrow \boxed{2\frac{35}{99}}$$