

Lesson 7.4:

Approximating Square Roots

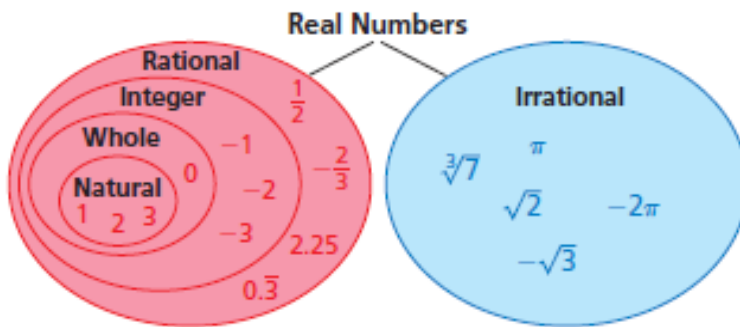
Essential Question

How can you find decimal approximations of square roots that are not rational?

Key Idea

Real Numbers

Rational numbers and irrational numbers together form the set of **real numbers**.



The square or cube root of any integer that is not a perfect square is irrational.

Irrational numbers neither terminate nor repeat.

Classify each real number.

- | | Number |
|----|--|
| a. | $\sqrt{12}$: Irrational |
| b. | -0.25 : Rational |
| c. | $-\sqrt{9} = -3$: Rational \rightarrow Integer |
| d. | $\frac{72}{4} = 18$: Rational \rightarrow Integer \rightarrow Whole # \rightarrow Natural # |
| e. | π : Irrational |

Classify the real number.

1. $0.121221222\dots$

Irrational

2. $-\sqrt{196} = -14$

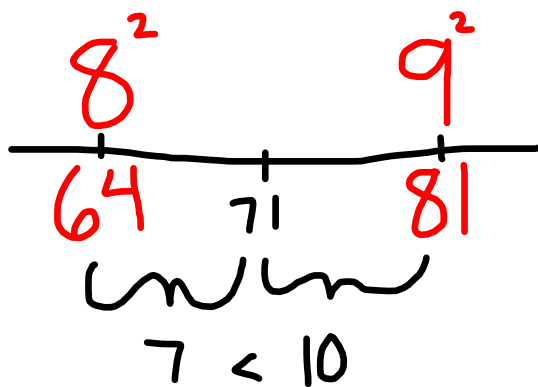
$$\begin{array}{r} 14 \\ \times 14 \\ \hline 56 \\ 140 \\ \hline 196 \end{array}$$

$R \Rightarrow \text{Integer}$

3. $\sqrt[3]{2}$

Irrational

Estimate $\sqrt{71}$ to the nearest integer.



$\sqrt{71}$ is bet.
8 and 9.

71 is closer to
64 than 81, so...

$\sqrt{71} \approx 8$

Estimate $\sqrt{71}$ to the nearest tenth.

Start by guessing

$$\begin{array}{r} 1 \\ 2 \\ 8.5 \\ \times 8.5 \\ \hline 425 \\ + 6800 \\ \hline 72.25 \end{array}$$

$$\begin{array}{r} 3 \\ 8.4 \\ \times 8.4 \\ \hline 336 \\ + 6720 \\ \hline 70.56 \end{array}$$

$$\begin{array}{r} 71.00 \\ - 70.56 \\ \hline .44 \end{array}$$

$$\begin{array}{r} 72.25 \\ - 71.00 \\ \hline 1.25 \end{array}$$

71 is closer to 70.56 than 72.25, so...

$$\sqrt{71} \approx \boxed{8.4}$$

Estimate the square root to the nearest (a) integer and (b) tenth.

4. $\sqrt{8}$ 2^2 $\boxed{3}$

$$\begin{array}{r} 2.9 \\ \times 2.9 \\ \hline 261 \\ + 580 \\ \hline 8.41 \end{array}$$

$$\begin{array}{r} 2.8 \\ \times 2.8 \\ \hline 224 \\ + 560 \\ \hline 7.84 \end{array}$$

5. $-\sqrt{13}$ -3^2 $\boxed{-4}$

$$\begin{array}{r} -3.8 \\ \times 3.8 \\ \hline 304 \\ + 1140 \\ \hline -14.44 \end{array}$$

$$\begin{array}{r} -3.7 \\ \times 3.7 \\ \hline 259 \\ + 1110 \\ \hline -13.69 \end{array}$$

$$\begin{array}{r} -3.6 \\ \times 3.6 \\ \hline 216 \\ + 1080 \\ \hline -12.96 \end{array}$$

6. $-\sqrt{24}$ -4^2 $\boxed{-5}$

$$\begin{array}{r} 4.9 \\ \times 4.9 \\ \hline 441 \\ + 1960 \\ \hline 24.01 \end{array}$$

7. $\sqrt{110}$ 10^2 11^2

$$\begin{array}{r} 10.5 \\ \times 10.5 \\ \hline 525 \\ + 10500 \\ \hline 110.25 \end{array}$$

$$\begin{array}{r} 10.4 \\ \times 10.4 \\ \hline 416 \\ + 10400 \\ \hline 108.16 \end{array}$$

Which is greater, $\sqrt{5}$ or $2\frac{2}{3}$?

guess

$$\begin{array}{r}
 2.3^2 \rightarrow 2.3 \\
 \times 2.3 \\
 \hline
 169 \\
 + 460 \\
 \hline
 5.29
 \end{array}$$

$\sqrt{5}$ is less than 2.3

2.3 is less than $2\frac{2}{3}$

$$\sqrt{5} < 2\frac{2}{3}$$

Which number is greater? Explain.

8. $4\frac{1}{5}\sqrt{23}$

↑

$$\begin{array}{r}
 4.2 \\
 \times 4.2 \\
 \hline
 84 \\
 + 1680 \\
 \hline
 17.64
 \end{array}$$

9. $\sqrt{10} - \sqrt{5}$

+

10. $-\sqrt{2} - 2$

$$-2 < -\sqrt{2} < -1$$

Order the numbers from least to greatest: $\sqrt{38}$, $\sqrt{\frac{100}{3}}$, 6.5

$$\sqrt{38} \approx 6$$

$$\sqrt{\frac{100}{3}} = \sqrt{33.\overline{3}} < 6$$

$$\begin{array}{r} 32 \\ \times 6.5 \\ \hline 325 \\ + 3900 \\ \hline 4225 \end{array}$$

$$\sqrt{\frac{100}{3}} < \sqrt{38} < 6.5$$