

# Lesson 1.3:

## Solving Equations with Variables on Both Sides

### Essential Question

How can you solve an equation that has variables on both sides?

 **Key Idea**
**Solving Equations with Variables on Both Sides**

To solve equations with variables on both sides, collect the variable terms on one side and the constant terms on the other side.

Same idea as before...combine like terms!

Solve  $15 - 2x = -7x$ . Check your solution.

$$\begin{array}{r|l}
 +2x & +2x \\
 \hline
 15 & = -5x \\
 \div -5 & \div -5 \\
 \hline
 -3 & = x
 \end{array}$$

$$15 - 2(-3) = -7(-3)$$

$$15 + (+6) = 21$$

$$21 = 21 \checkmark$$

Solve  $-2(x - 5) = 6\left(2 - \frac{1}{2}x\right)$ .

$$\begin{array}{r|l} -2x + 10 & = 12 - 3x \\ +3x & +3x \\ \hline x + 10 & = 12 \\ -10 & -10 \\ \hline x & = 2 \end{array}$$

$$\begin{aligned} -2(2-5) &= 6\left(2 - \frac{1}{2}(2)\right) \\ -2(-3) &= 6(2-1) \\ 6 &= 6(1) \\ 6 &= 6 \checkmark \end{aligned}$$

Solve the equation. Check your solution.

1.  $-3x = 2x + 19$

$$\begin{array}{r|l} -2x & -2x \\ \hline -5x & = 19 \\ \div -5 & \div -5 \\ \hline x & = -\frac{19}{5} \end{array}$$

$$-3\left(-\frac{19}{5}\right) = 2\left(-\frac{19}{5}\right) + 19$$

$$\frac{57}{5} = -\frac{38}{5} + 19$$

$$\frac{95}{5} = 19 \checkmark$$

2.  $2.5y + 6 = 4.5y - 1$

$$\begin{array}{r|l} -4.5y & -4.5y \\ \hline -2y + 6 & = -1 \\ -6 & -6 \\ \hline -2y & = -7 \\ \div -2 & \div -2 \\ \hline y & = \frac{7}{2} \end{array}$$

$$2.5\left(\frac{7}{2}\right) + 6 = 4.5\left(\frac{7}{2}\right) - 1$$

$$8.75 + 6 = 15.75 - 1$$

$$14.75 = 14.75 \checkmark$$

3.  $6(4 - z) = 2z$

$$\begin{array}{r|l} 24 - 6z & = 2z \\ +6z & +6z \\ \hline 24 & = 8z \\ \div 8 & \div 8 \\ \hline 3 & = z \end{array}$$

$$6(4-3) = 2(3)$$

$$6(1) = 6 \checkmark$$

Solve  $3 - 4x = -7 - 4x$ .

$$\begin{array}{r} +4x \quad +4x \\ \hline 3 \quad = \quad -7 \end{array}$$

OH NO. This means that this equation has **no solution**.

Solve  $6x + 4 = 4\left(\frac{3}{2}x + 1\right)$ .

$$6x + 4 = 6x + 4$$

WOAH. Both sides are exactly the same!  
This means that the equation has **infinitely many solutions** since any value of  $x$  would work.

Solve the equation.

4.  $2x + 1 = 2x - 1$

$$\begin{array}{r} -2x \quad | \quad -2x \\ \hline 1 = -1 \end{array}$$

NO solution

5.  $\frac{1}{2}(6t - 4) = 3t - 2$

$$3t - 2 = 3t - 2$$

infinitely many solutions

6.  $\frac{1}{3}(2b + 9) = \frac{2}{3}\left(b + \frac{9}{2}\right)$

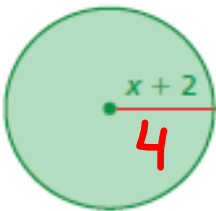
$$\frac{2}{3}b + 3 = \frac{2}{3}b + \frac{2}{3} \cdot \frac{9}{2}$$

$$\frac{2}{3}b + 3 = \frac{2}{3}b + 3 \text{ IMS}$$

7.  $6(5 - 2v) = -4(3v + 1)$

$$\begin{array}{r} 30 - 12v \neq -12v - 4 \\ +12v \quad | \quad +12v \\ \hline 30 \neq -4 \\ \text{NS} \end{array}$$

The circles are identical. What is the area of each circle?

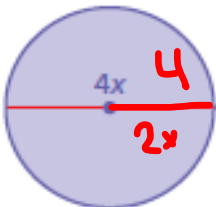


(A) 2

(B) 4

(C)  $16\pi$

(D)  $64\pi$



$$\begin{array}{r} x + 2 \neq 2x \\ -x \quad | \quad -x \\ \hline 2 \neq x \end{array}$$

$$\begin{aligned} A &= \pi r^2 \\ &= \pi 4^2 \\ &= 16\pi \end{aligned}$$

A boat travels  $x$  miles per hour upstream on the Mississippi River. On the return trip, the boat travels 2 miles per hour faster. How far does the boat travel upstream?



$$x \frac{\text{mi}}{\text{hr}} \cdot 3 \text{ hr} = 3x \text{ mi}$$

$$(x-2) \frac{\text{mi}}{\text{hr}} \cdot 2.5 \text{ hr} = 2.5(x-2) \text{ mi}$$

$$3x = 2.5(x-2)$$

$$3x = 2.5x - 5$$

$$\begin{array}{r} 3x \\ -2.5x \\ \hline .5x = -5 \end{array}$$

$$.5x = -5$$

$$\begin{array}{r} .5x \\ \div .5 \\ \hline x = -10 \end{array}$$

$$x = -10$$

$$3(-10)$$

$$= -30 \text{ mi}$$

8. **WHAT IF?** In Example 5, the diameter of the purple circle is  $3x$ . What is the area of each circle?

$$1.5x = x + 2$$

$$-x \quad -x$$

$$.5x = 2$$

$$x = 4$$

$$r = 4 + 2 = 6$$

$$\pi r^2$$

$$\pi 6^2$$

$$36\pi$$

9. A boat travels  $x$  miles per hour from one island to another island in 2.5 hours. The boat travels 5 miles per hour faster on the return trip of 2 hours. What is the distance between the islands?

$$x \frac{\text{mi}}{\text{hr}} \cdot 2.5 \text{ hr} = 2.5x \text{ mi}$$

$$(x+5) \frac{\text{mi}}{\text{hr}} \cdot 2 \text{ hr} = 2(x+5) \text{ mi}$$

$$2.5x = 2(x+5)$$

$$2.5x = 2x + 10$$

$$.5x = 10$$

$$x = 20$$

$$2.5(20)$$

$$= \boxed{50 \text{ mi}}$$